Séminaire N. Bourbaki

SAMEDI 30 MARS 2019

Institut Henri Poincaré (amphi. Hermite)
11 rue Pierre et Marie Curie, 75005 Paris

11h00 Beatrice POZZETTI
Higher rank Teichmüller theories

Let Γ be the fundamental group of a compact surface S with negative Euler characteristic, and G denote $\mathrm{PSL}(2, \mathbf{R})$, the group of isometries of the hyperbolic plane. Goldman observed that the Teichmüller space, the parameter space of marked complex structures on S can be identified with a connected component of the character variety $\mathrm{Hom}(\Gamma, G)/G$, which can be selected by means of a characteristic invariant. Thanks to the work of Labourie, Burger-Iozzi-Wienhard, Fock-Goncharov, Guichard-Wienhard we now know that, surprisingly, this is a much more general phenomenon: there are many higher rank semisimple Lie groups G admitting components of the character variety only consisting of injective homomorphisms with discrete image, the socalled higher Teichmüller theories. The richness of these theories is partially due to the fact that, as for the Teichmüller space, truly different techniques can be used to study them: bounded cohomology, Higgs bundles, positivity, harmonic maps, incidence structures, geodesic currents, real algebraic geometry... In my talk I will overview a number of recent results in the field (following Labourie, Burger-Iozzi-Wienhard, Fock-Goncharov, Guichard-Wienhard, Bonahon-Dreyer, Li, Zhang, Martone-Zhang, Bargaglia, Alessandrini-Li, Collier-Tholozan-Toulisse.)

14h30 Luca MIGLIORINI
HOMFLY polynomials from the Hilbert schemes of a planar curve, *after D. Maulik, A. Oblomkov, V. Shende...*

Among the most interesting invariants one can associate with a link $\mathcal{L} \subset S^3$ is its HOMFLY polynomial $P(\mathcal{L}, v, s) \in \mathbb{Z}(v^{\pm 1}, (s - s^{-1})^{\pm 1})$. A. Oblomkov and V. Shende conjectured that this polynomial can be expressed in algebraic geometric terms when $\mathcal{L}$ is obtained as the intersection of a plane curve singularity $(C, p) \subset \mathbb{C}^2$ with a small sphere centered at p: if $f = 0$ is the local equation of C, its Hilbert scheme $C_p^{(n)}$ is the algebraic variety whose points are the length n subschemes of C supported at p, or, equivalently, the ideals $I \subset \mathbb{C}((x, y))$ containing f and such that $\dim \mathbb{C}((x, y))/I = n$. If $m : C_p^{(n)} \to \mathbb{Z}$ is the function associating with the ideal I the minimal number $m(I)$ of its generators, they conjecture that the generating function $Z(C, v, s) = \sum_n s^{2n} \int_{C_p^{(n)}} (1 - v^2)^{m(I)} d\chi(I)$ coincides, up to a renormalization, with $P(\mathcal{L}, v, s)$. In the formula the integral is done with respect to the Euler characteristic measure $d\chi$. A more refined version of this surprising identity, involving a "colored" variant of $P(\mathcal{L}, v, s)$, was conjectured to hold by E. Diaconescu, Z. Hua and Y. Soibelman. The seminar will illustrate the techniques used by D. Maulik to prove this conjecture.

16h00 Adam HARPER
The Riemann zeta function in short intervals

A classical idea for studying the behaviour of complicated functions, like the Riemann zeta function $\zeta(s)$, is to investigate averages of them. For example, the integrals over $T \leq t \leq 2T$ of various powers of $\zeta(1/2 + it)$, sometimes multiplied by some other cleverly chosen function, have been investigated extensively to deduce upper and lower bounds for the maximum size of $\zeta(1/2 + it)$. More recently, Fyodorov and Keating have proposed the investigation of much shorter integrals over $T \leq t \leq T + 1$. This turns out to lead to interesting connections between various issues in number theory, analysis, mathematical physics and probability, such as branching random walk and multiplicative chaos. I will try to explain some of these connections, ideas from the proofs, and what they tell us about the zeta function.